Enhanced NSGA Based on Adaptive Crossover Rate and Reference Points

Ima O. Essiet†, Yanxia Sun‡
Department of Electrical and Electronics Engineering Science
University of Johannesburg
South Africa
imaessiet82@gmail.com†, ysun@uj.ac.za‡

Zenghui Wang
Department of Electrical and Mining Engineering
University of South Africa
South Africa
wangz:@unisa.ac.za

***Abstract*—The challenges of many-objective optimization are investigated; and one new algorithm, which is based on the NSGA-II, is proposed for multi-objective optimization in this paper. The reference points and an adaptable crossover rate are combined in the algorithm to improve the performance of NSGA-II. The performance of NSGA for optimizing the many objective search space is examined with and without the proposed algorithm through a constrained two-objective problem with up to 40 dimensions. Simulation results show that the proposed algorithm improves the performance of NSGA for the selected test problem in generations where a non-dominated set is not obtained by 39%.**

Keywords-optimization; inverted generational distance; reference points; convergence; diversity

I. INTRODUCTION

Many problems occurring in engineering today are modeled with many objectives. These objectives are often optimized subject to certain constraints, which gives rise to mathematical and computational challenges. This is because when the solution space is controlled by many objectives, it becomes difficult to find a set of suitable solutions that will guide the population to Pareto optimality. This is one of the reasons why the field of multi-objective optimization is still in its infancy [1].

A balance must be achieved between convergence and diversity with respect to multi-objective optimization. There are two important attributes associated with the Pareto front: its shape and continuity [2]. These attributes determine the level of difficulty for an optimization technique to obtain the Pareto optimal set of solutions to a particular problem. Metaheuristics are well suited to handle both aspects of the Pareto front because they are efficient in terms of their ability to quickly generate the Pareto optimal set at once; also they are less sensitive to the shape and continuity of the Pareto front compared to mathematical programming methods. The non-dominated sorting genetic algorithm (NSGA) is one of several metaheuristic-based optimization techniques which have been used to successfully optimize multi-objective problems [3-4].

This paper uses the NSGA-II technique to observe the behaviour of the Pareto front for a problem consisting of two objectives, two constraints and a small feasible region. The dimensionality of the solution space and the crossover rate of the NSGA algorithm are varied iteratively, and the Pareto front of the selected problem is discontinuous. An algorithm called reference point-based NSGA (RP-NSGA) is also proposed to improve the performance of NSGA-II for constrained multi-objective optimization.

II. OVERVIEW OF THE NSGA-II ALGORITHM

The NSGA-II algorithm is based on the principle of non-dominance, in which a solution S_A is said to dominate another S_B if its objective function (OF) is overall better than that of S_B. A collection of non-dominated solutions forms the Pareto optimal set. For the NSGA-II algorithm, one of three conditions must be satisfied if a solution A is to constrained-dominate another solution B [3]:

- Solution A is feasible, and solution B is not
- Both solution A and B are infeasible, but solution A has a smaller constraint violation
- Both solution A and B are feasible and solution A dominates solution B

The NSGA-II algorithm provides suitable solutions to multi-objective optimization problems with up to three (3) objectives. However, for problems involving more than three objectives (known as many-objective problems) as well as problems with high dimensionality, the performance of NSGA-II degrades significantly [5].

The crossover rate (CR) is another important metric with regard to the performance of evolutionary algorithms including NSGA-II [3]. Its value is selected between 0 and 1 to guide the convergence of the solutions in the OF and decision variable space towards the Pareto front.

Several recent attempts have been made to realize optimizers to solve problems with many objectives. These attempts have led to a number of challenges [5]. Based on these challenges, a number of remedial approaches have been proposed in literature [6-8]. One of these involves the refinement of the concept of dominance among the potential solutions in such a way that optimal convergence to the Pareto optimal front is enhanced viz-a-viz the selection pressure [9-10]. Another approach involves using performance metrics during the optimization process to guide the search for Pareto optimal solutions. Those solutions which perform better in terms of convergence and diversity are selected [11-12]. A third approach tackles the issue of convergence by using convergence metrics to select non-dominated solutions. This is done by increasing selection pressure to the most promising solutions in relation their proximity to the Pareto front [13-

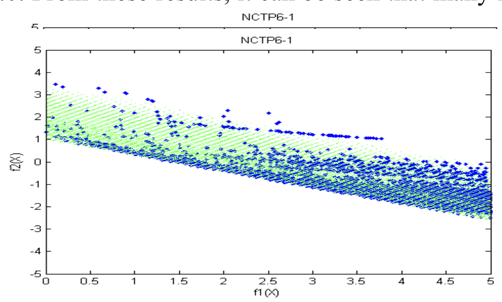
14]. The approach used in this paper is a combination of the use of both performance and convergence metrics to obtain the Pareto optimal set.

A. *Demonstrating the Challenge of Diversity and Convergence of Non-dominated Solutions at High Dimension*

In this section, a 2-objective problem is simulated with 30 and 40 dimensions to represent a scenario similar to a many-objective optimization problem. The effect of high dimensionality on crossover rate and Pareto front behaviour is examined with respect to NSGA-II. Its performance is evaluated using the inverted generational distance (IGD) metric. The test problems used in this paper are from [15].

The size of the feasible region for all test problems is small and the topology of the Pareto front is determined by six parameters according to [15]. The NSGA-II algorithm is modified to handle the two problem constraints based on the adaptive tradeoff model (ATM) also proposed in [15]. The dimensionality of the problem is determined by the matrix describing the problem. The inverted generational distance (IGD) is used to measure an algorithm's ability to optimize a multi-objective problem by measuring its convergence towards the Pareto front. The smaller the IGD value, the better an algorithm's performance [16].

Fig. 1 examines the effect of crossover rate variation on the behaviour of the Pareto front. It can be observed that at lower values of crossover rate, the convergence of non-dominated solutions is almost non-existent. However, as the crossover rate increases to between 0.8 and 0.9, convergence of non-dominated solutions toward the Pareto front can be clearly observed. From Fig. 3, it can be observed that the average IGD values over the 100 iterations are highest at n=40 compared with those at n=30. In general, the IGD values increase as the dimension of the search space increases. At n=40, at least 21 IGD values are between 2.0 and 4.4. Also, for the case of n=30, the highest IGD value is about 3.0. From these results, it can be seen that many more



solutions try to converge to the Pareto front as the dimension of the search space increases. This explains why there is a superimposition of multiple Pareto fronts at n=40 compared to n=30. While there are so many possible solutions in search space of high dimension, it becomes more difficult to obtain a distinct Pareto front within the feasible region. This inference is further confirmed by the number of iterations with IGD values of 0.0 in the IGD plot at n=40. There are 21 null IGD values compared with 13 null values for n=30.

III. Proposed Algorithm to Enhance Diversity and Convergence

The IGD values are plotted graphically for 100 iterations across 10 generations of the NSGA algorithm. The algorithm is as presented in Algorithm 1.

ALGORITHM 1: RP-NSGA Algorithm

Enter Numiter, CR, maxGen, theta, d_1, d_2, nPop

$$H = \binom{M+p-1}{p}, \theta = [5.0 : 1.0, -1.0], CR = [0.80 : 0.90, 0.01], PBI = d_1 + \theta d_2$$

If *IGD=null then*

  increase CR in steps of 0.01

  compute PBI, H

  else if CR=0.9 then

  decrease theta in steps of 1.0

  compute PBI

  end

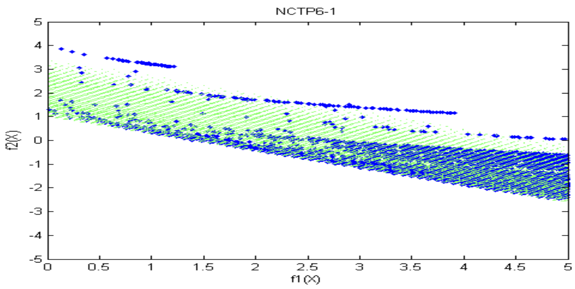
end

Use non-dominated sorting to divide maxGen into several non-dominated levels

end

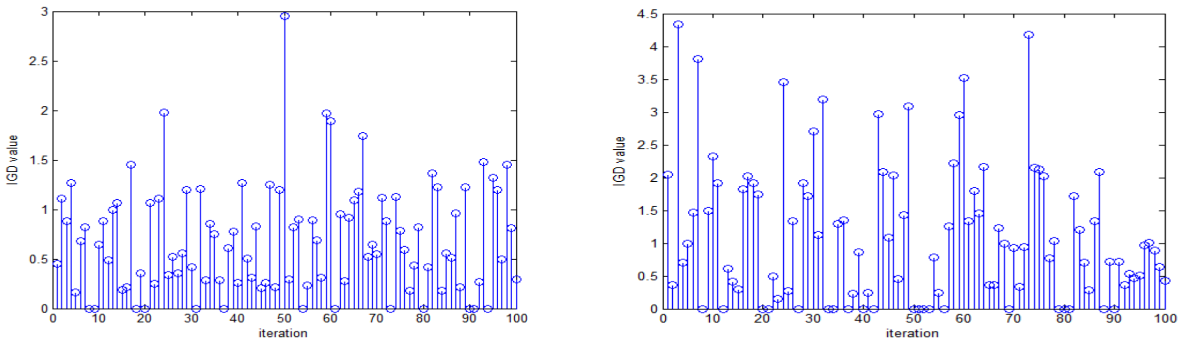
(a)

(b)



(c)

Figure 1. Effect of Crossover rate (CR) on Pareto front behaviour (a) CR=0.5 (b) CR=0.8 and (c) CR=0.9.



(a) (b)

Figure 2. IGD values for 100 iterations for (a) n=30 and (b) n=40.

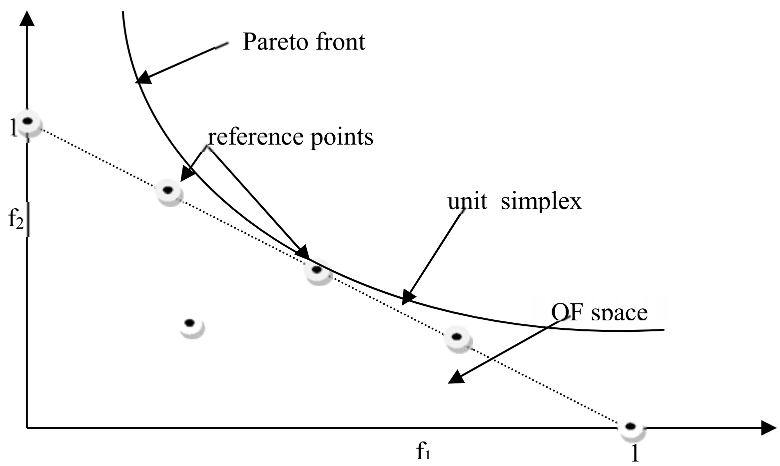


Figure 3. Placement of reference points along unit simplex and within feasible region.

As seen from the algorithm (called RP-NSGA), Das and Dennis's method [17] is used to specify the number of reference points (H), which guide the movement of the solutions in the OF space toward the Pareto front. The number of points is user-defined according to the following relation:

$$H = \binom{M + p - 1}{p} \qquad (1)$$

where
M= number of objectives
p= number of division along the unit line
H= number of reference points

For a 2-objective OF space, the simplex plane will be a triangle with apex at (1,0) and (0,1) respectively as depicted in Fig. 3. The number of reference points along the unit simplex was selected as five in this paper. Therefore, the number of reference points is 6.

The reference points are positioned as shown in Fig 3 The penalty-based boundary intersection method (PBI) [18] is employed to penalize weight vectors based on convergence and diversity. The relation is as shown in equation (2):

$$PBI(x_1, x_2) = d_1 + \theta d_2 \tag{2}$$

From the equation, d_1 controls solution convergence by moving solutions towards the Pareto front. d_2 spreads solutions along the Pareto front, thus controlling diversity of the non-dominated solutions. Θ is a user defined metric whose value can be greater than or equal to zero. Research has shown that varying Θ changes the search pattern for non-dominated solutions [8].

IV. RESULTS AND DISCUSSION

The inverted generational distance (IGD) metric was used to measure the performance of RP-NSGA. The results obtained were then compared with those of NSGA-II. IGD values over 100 iterations for the same constrained problem proposed by Wang et al [15] were used. Results are shown in Fig. 4 for 30 and 40-dimensional problems respectively.

Compared to the IGD values obtained in Fig. 2, there is a 39% reduction in the worst IGD value for n=30 when RP-NSGA is applied, and a 19% reduction when n=40. However, it was observed that the number of null (zero) IGD values reduces when the algorithm is applied for both n=30 and n=40 by 4% and 7% respectively. This demonstrates that RP-NSGA is likely to improve the ability of NSGA-II to obtain solutions for the Pareto optimal front at high dimensions and with many objectives.

Table 1 presents the best, median and worst IGD values for 30 and 40 dimensions of the selected constrained problem with and without RP-NSGA. There is no significant improvement for the best obtained values when RP-NSGA is applied. However, there is significant improvement in IGD values for the median and worst values (25% and 9% respectively for n=30, and 39% and 21% improvement for n=40). This is likely due to the fact that the algorithm is concerned with cases where null IGD values are obtained, which is when the distance of the solutions from the Pareto These results demonstrate that adaptively varying the crossover rate while imposing boundary violation penalty improves the performance of NSGA.

optimal front is very large. These results therefore demonstrate the challenge of many-objective optimization, because improvement with regard to convergence and diversity is still marginal at high dimensions of the solution space.

The average standard deviation over 100 iterations for NSGA with and without reference points is also shown in Table 1. It is observed that the error is generally higher in NSGA without reference points with respect to the Pareto front points. This means that the reference point approach improves the quality of solutions with respect to their proximity to the Pareto optimal solution.

From Fig. 5, it is observed that the computational time linearly increases over the 100 iterations for 30 and 40-dimensional problems. The computational time is slightly higher for n=40 between 30 and 80 iterations; this is likely due to increasing the number of possible solutions which adds to the computational overhead. The IGD value for the 100 iterations take 639 and 644 seconds to compute for n=30 and n=40 respectively. Total computational time for n=30 was 8.3 hours and n=40 took 8.5 hours.

V. CONCLUSIONS

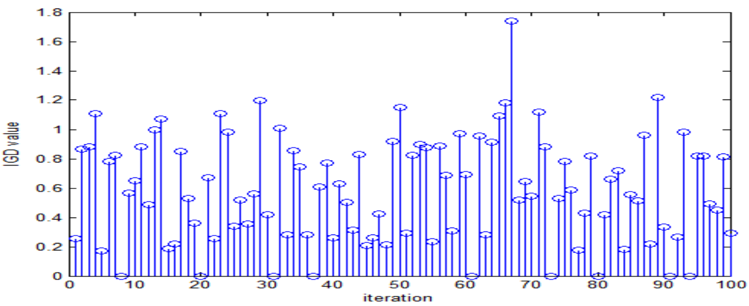
This paper has examined the challenges of NSGA-II in the optimization of high dimensional problems. These challenges majorly involve slow convergence to the Pareto front and proper diversity among the non-dominated solutions. A high dimensional scenario has been created with 2 objective functions and up to a 40-dimensional problem with constraints. Several attempts to remedy the problems of many-objective optimization have also been discussed and some have yielded very promising results.

An algorithm called RP-NSGA has been proposed in this paper which uses reference points and an adaptive crossover rate to improve the performance of NSGA-II when null IGD values are obtained for generations of non-dominated solutions. Results have shown that the performance of NSGA-II improves by 39% for the worst IGD value over an average of 100 iterations for RP-NSGA compared to the original NSGA.

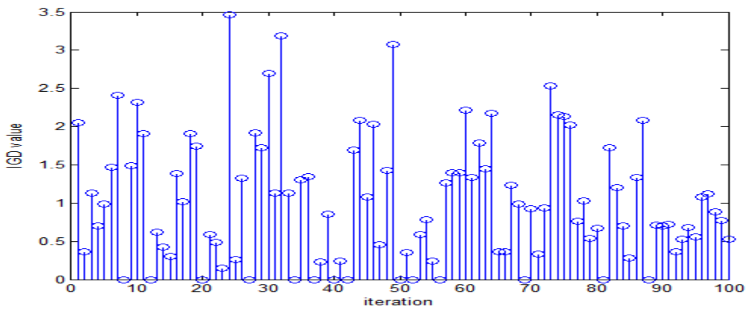
Future work will investigate a possible relationship between crossover rate and placement of reference points within the search space.

TABLE I. BEST, MEDIAN, WORST IGD VALUES AND AVERAGE STANDARD DEVIATION OVER 100 ITERATIONS FOR N=30 AND N=40 FOR NSGA WITH AND WITHOUT REFERENCE POINTS

	Dimension	Best	Median	Worst	Average Std. Dev.
With RP-NSGA	30	0.20	1.20	2.90	1.37E-3
	40	0.20	2.30	4.40	2.13E-2
Without RP-NSGA	30	0.18	0.90	1.78	3.68E-2
	40	0.20	1.80	3.49	2.88E-1



(a)



(b)

Figure 4. IGD values for (a) n=30 and (b) n=40 for 100 iterations with RP-NSGA algorithm.

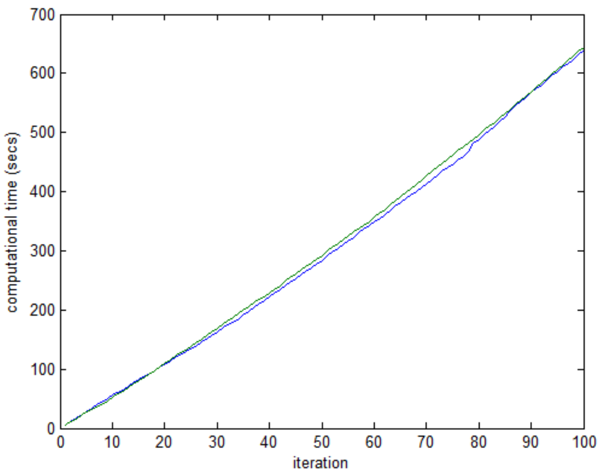


Figure 5. Computational time over 100 iterations for n=30 (blue) and n=40 (green).

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